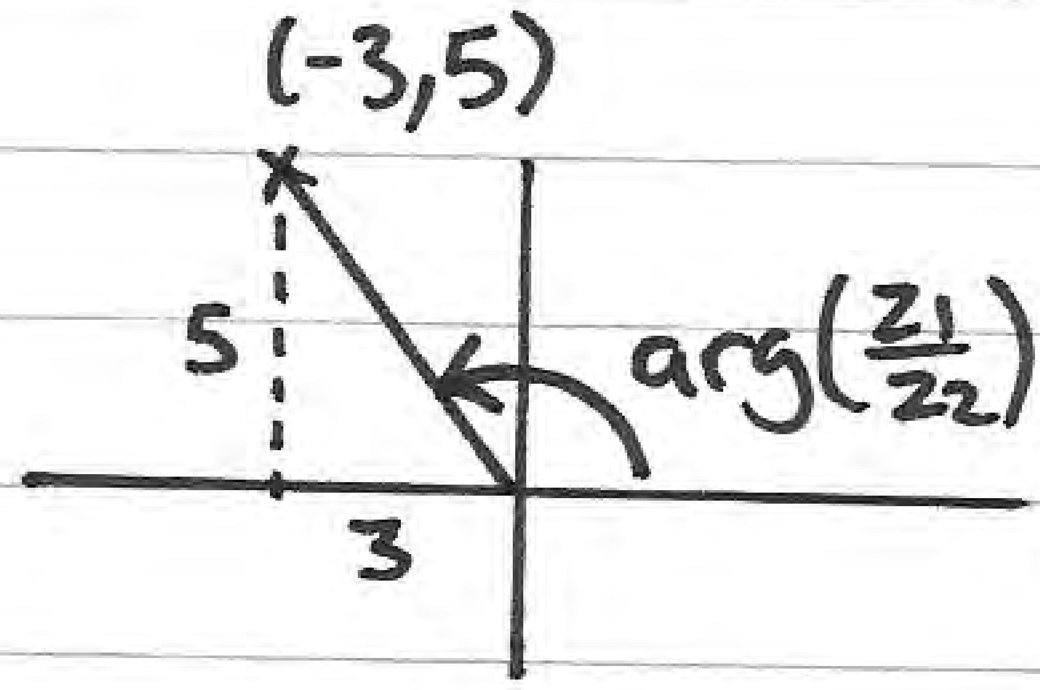
FP1 JAN 2010

1) $\frac{z_1}{z_2} = \frac{2+8i}{1-i}\frac{(1+i)}{(1+i)} = \frac{2+2i+8i+8i^2}{1-i^2} = \frac{-6+10}{2}$

$\frac{z_1}{z_2} = \underline{-3+5i}$

b) $\left|\frac{z_1}{z_2}\right| = \sqrt{3^2+5^2} = \sqrt{34}$

c)



$\arg\left(\frac{z_1}{z_2}\right) = \pi - \tan^{-1}\left(\frac{5}{3}\right)$

$= \underline{2.11^c}$

2) $f(x) = 3x^2 - \frac{11}{x^2}$

$f(1.3) = -1.439$
$f(1.4) = 0.268$

b)

a	$f(a)$	b	$f(b)$	$\frac{a+b}{2}$	$f\left(\frac{a+b}{2}\right)$
1.3	-1.4	1.4	0.3	1.35	-0.57
1.35	-0.6	1.4	0.3	1.375	-0.15
1.375	-0.15	1.4	0.3		

$\Rightarrow \quad \alpha \in [1.375, 1.4]$

c) $f'(x) = 6x + 22x^{-3}$

$x_1 = 1.4 - \frac{f(1.4)}{f'(1.4)} = 1.384$

3) $n=1 \quad u_1 = 2 \qquad u_1 = 5^{1-1}+1 = 1+1 = 2$ $\therefore$ true for $n=1$

$n=2 \quad u_2 = 5(2)-4 = 6 \qquad u_2 = 5^{2-1}+1 = 5+1 = 6$ $\therefore$ true for $n=2$

$n=k \qquad u_k = 5(u_{k-1})-4 \qquad u_k = 5^{k-1}+1$ assume true

$n=k+1 \qquad u_{k+1} = 5(u_k)-4 = 5(5^{k-1}+1)-4$
$$= 5^k + 5 - 4$$
$$= 5^k + 1$$

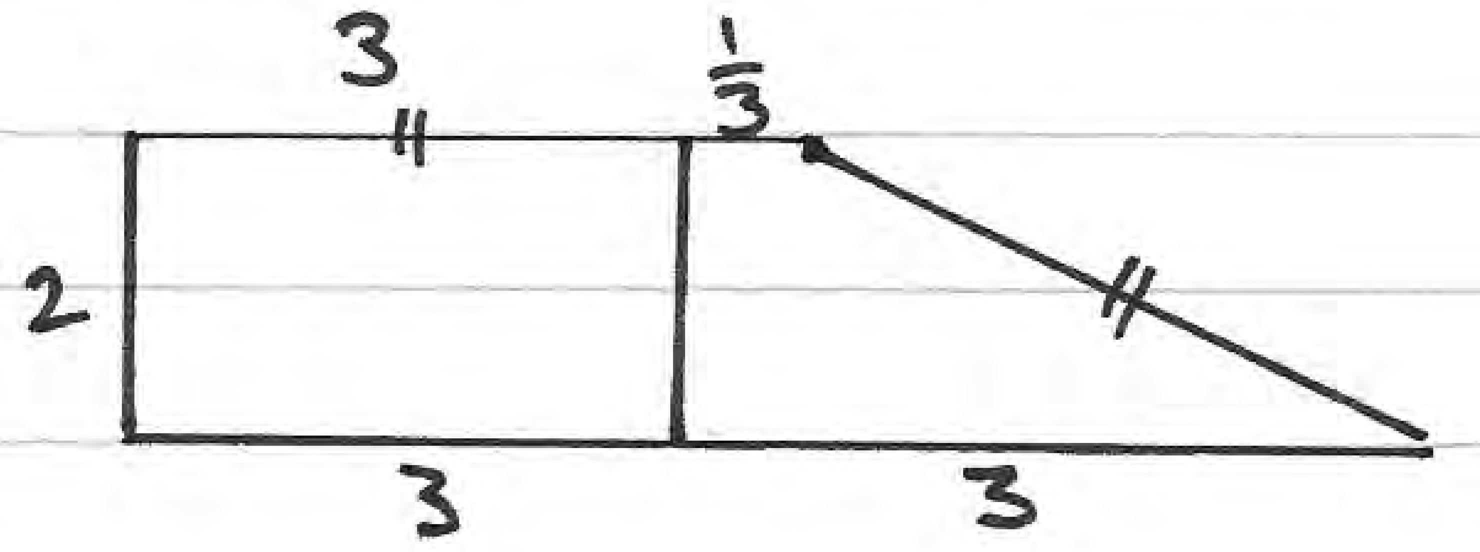
$u_{k+1} = 5^{(k+1)-1}+1 = 5^k+1$ as required.

true for $n=1$, true for $n=k+1$ if true for $n=k$
$\therefore$ by Induction true for all $n \in \mathbb{Z}^+$

4) $y^2 = 12x = 4ax \Rightarrow a=3$ focus $(3,0)$ directrix $x+3=0$

a) $S(3,0)$

b) $A(-3,0)$



$x = \frac{1}{3} \Rightarrow y^2 = 4$
$\Rightarrow P(\frac{1}{3}, 2) \Rightarrow B(-3,2)$

Perimeter $= 2\times(3\frac{1}{3})+2+6 = \frac{44}{3}$

5) $A = \begin{pmatrix} a & -5 \\ 2 & a+4 \end{pmatrix} \qquad \det A = a(a+4)+10 = a^2+4a+10$

b) $\det A = (a+2)^2 - 4 + 10 = (a+2)^2+6 \Rightarrow \det A \geq 6$
hence non-singular

c) $a=0 \quad A = \begin{pmatrix} 0 & -5 \\ 2 & 4 \end{pmatrix} \quad \det A = 10 \quad A^{-1} = \frac{1}{10}\begin{pmatrix} 4 & 5 \\ -2 & 0 \end{pmatrix}$

6) $5-2i$

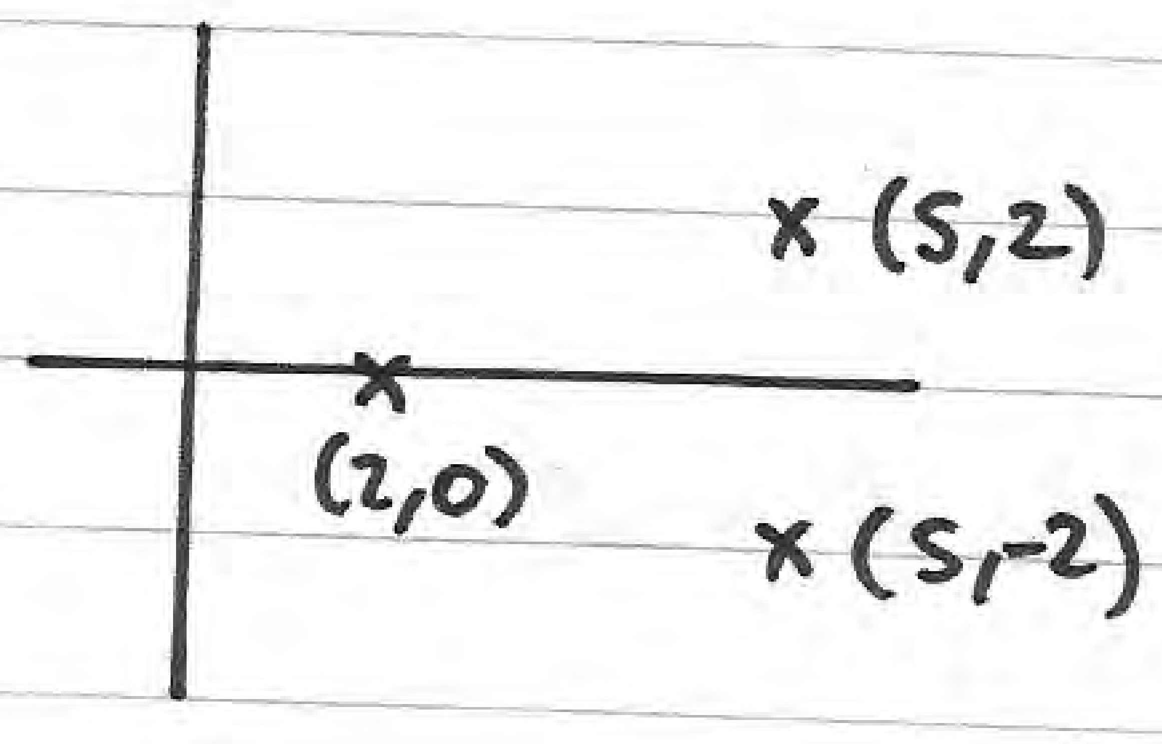
b) $(x-2)(x-(5+2i))(x-(5-2i))$

Sum $=10$
product $= 25-4i^2$
$= 29$

$\Rightarrow (x-2)(x^2-10x+29)$

$d = -2\times 29 = \underline{-58}$ $\quad c = -2\times -10 + 29 = \underline{49}$

c)



7) $y = c^2x^{-1} \Rightarrow \dfrac{dy}{dx} = -c^2x^{-2} = \dfrac{-c^2}{x^2} = \dfrac{-c^2}{c^2t^2} = -\dfrac{1}{t^2}$

$\Rightarrow m_t = -\dfrac{1}{t^2} \quad \Rightarrow y - \dfrac{c}{t} = -\dfrac{1}{t^2}(x-ct)$

$\times t^2 \quad yt^2 - ct = -x + ct$

$\Rightarrow t^2y + x = 2ct$ #

b) $(15c, -c) \Rightarrow t^2(-c) + (15c) = 2ct$

$\Rightarrow \cancel{c}t^2 + 2\cancel{c}t - 15\cancel{c} = 0$

$\Rightarrow t^2 + 2t - 15 = 0$

$\Rightarrow (t+5)(t-3) = 0 \quad t=-5, t=3$

$\left(ct, \dfrac{c}{t}\right) \Rightarrow \underline{A\left(-5c, -\dfrac{c}{5}\right)} \quad \underline{B\left(3c, \dfrac{c}{3}\right)}$

8) $n=1 \quad \sum_{1}^{1} r^3 = 1^3 = 1 \qquad \frac{1}{4}(1)^2(1+1)^2 = \frac{1}{4}\times 1 \times 2^2 = 1 \therefore$ tr

$n=2 \quad \sum_{1}^{2} r^3 = 1^3 + 2^3 = 9 \qquad \frac{1}{4}(2)^2(2+1)^2 = \frac{1}{4}\times 4 \times 9 = 9 \therefore$ true

$n=k \quad \sum_{1}^{k} r^3 = \frac{1}{4}k^2(k+1)^2$ assume true

$n=k+1 \quad \sum_{1}^{k+1} r^3 = \frac{1}{4}(k+1)^2(k+2)^2$

$$\sum_{1}^{k+1} r^3 = \sum_{1}^{k} r^3 + (k+1)^3 = \frac{1}{4}k^2(k+1)^2 + (k+1)^3$$

$$= \frac{1}{4}(k+1)^2\left(k^2 + 4(k+1)\right)$$

$$= \frac{1}{4}(k+1)^2(k^2+4k+4)$$

$$= \frac{1}{4}(k+1)^2(k+2)^2$$ as required.

true for $n=1$, true for $n=k+1$ if true for $n=k$
$\therefore$ true for all $n \in \mathbb{Z}^+$ by Induction.

b) $\sum_{r=1}^{n} r^3+3r+2 = \sum_{r=1}^{n} r^3 + 3\sum_{r=1}^{n} r + \sum_{r=1}^{n} 2$

$= \frac{1}{4}n^2(n+1)^2 + 3\left(\frac{1}{2}n(n+1)\right) + 2n$

$= \frac{1}{4}n\left[n(n+1)^2 + 6n(n+1) + 8\right]$

$= \frac{1}{4}n(n^3+2n^2+n+6n+6+8]$

$= \frac{1}{4}n(n^3+2n^2+7n+14)$

$= \frac{1}{4}n(n^2(n+2)+7(n+2))$

$= \frac{1}{4}n(n+2)(n^2+7)$ #

c) $\sum_{15}^{25} r^3+3r+2 = \frac{1}{4}(25)(27)(25^2+7) - \frac{1}{4}(14)(16)(14^2+7)$

$= 95282$

9) $M = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ rotation 45° anticlockwise about the origin

b) $\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} 3\sqrt{2} \\ 4\sqrt{2} \end{pmatrix}$

$\Rightarrow \frac{1}{\sqrt{2}}p - \frac{1}{\sqrt{2}}q = 3\sqrt{2}$ $\quad \times\sqrt{2} \quad p-q=6$

$+$

$\Rightarrow \frac{1}{\sqrt{2}}p + \frac{1}{\sqrt{2}}q = 4\sqrt{2}$ $\quad \times\sqrt{2} \quad p+q=8$

$2p=14$

$p=7 \quad q=1$

c) $OA = \sqrt{7^2+1^2} = \sqrt{50} = \underline{5\sqrt{2}}$

d) $M^2 = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

e) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 3\sqrt{2} \\ 4\sqrt{2} \end{pmatrix} = \begin{pmatrix} -4\sqrt{2} \\ 3\sqrt{2} \end{pmatrix}$

$\underline{C(-4\sqrt{2}, 3\sqrt{2})}$